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Choose Wisely: Static or Kinetic Friction—The Power of Dimensionless Plots

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Consider a problem of sliding blocks, one stacked atop the other, resting on a frictionless table. If the bottom block is pulled horizontally, nature makes a choice: if the applied force is small, static friction between the blocks accelerates the blocks together, but with a large force the blocks slide apart. In that case, kinetic friction still forces the upper block forward but with less acceleration than the lower block. The choice, then, lies in the relative terms—what is meant by small and large? After a confusing experience during a recent exam, we've found a demonstration and graphical presentation that can help clarify the distinction between static and kinetic friction.

The Exam Problem

This investigation started when we modified parameters in a textbook[1] problem in order to use it in an exam. As in Fig. 1, block m_2 sits atop a larger block m_1 on a frictionless tabletop. Block m_1 is pulled to the right with a force F. The coefficients of friction between the blocks are μ_s and μ_k for the static and kinetic cases, respectively, but students were not told which case to apply. For the exam, we used $F = 9.0$ N, $m_2 = 2.0$ kg, $m_1 = 4.0$ kg, $\mu_s = 0.30$, and $\mu_k = 0.20$. Kinetic friction was appropriate for the original parameter set, as well as an in-class example. Even our top students didn't pause to consider the possibility of static friction for this parameter set. With the assumption of kinetic friction, they found the top block to accelerate *faster* than the bottom one! Those thoughtful students were troubled by this counterintuitive result and sought to learn more about the choice made by nature between static and kinetic friction.

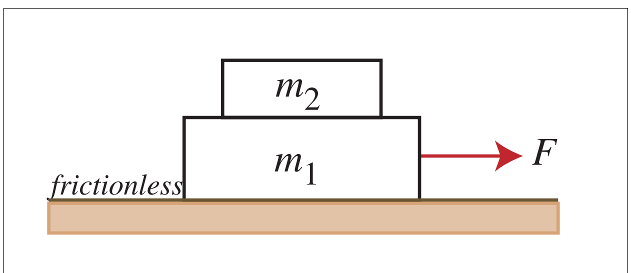

Fig. 1. The problem of two blocks. There is friction between them, but they rest on a frictionless table. The force on the lower block is horizontal.

The Demonstration and Dimensionless Plot

With the assistance of several curious students, we designed an experiment to show the transition from static to kinetic friction. An air table provided a nearly frictionless surface, blocks of MDF (medium density fiberboard, 1/2 in thick) served as the masses, and a string, pulley, and hanging weight applied the force. To record acceleration, we performed frame-by-frame video analysis (30 f/s) to track the position of the blocks as viewed from the side. Acceleration was determined as the slope of velocity-versus-time graphs with 95% confidence intervals in the neighborhood of ± 5 to 10%. The masses of the blocks were 0.063 kg and 0.157 kg, but the coefficients of friction were unknown. Three trials at each of 12 different hanging weights spanned the transition between static

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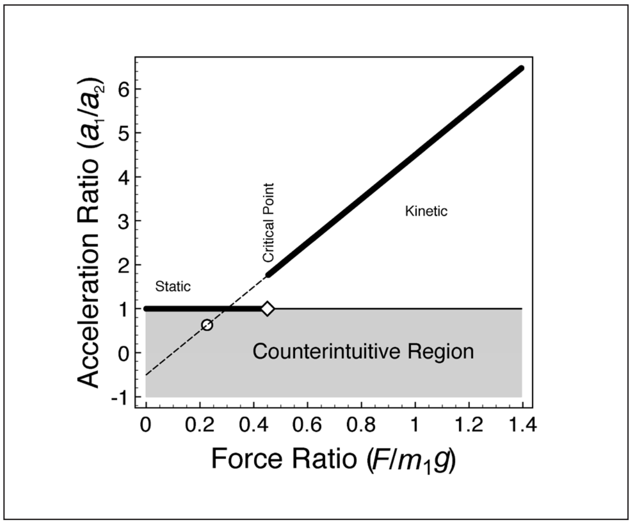

Fig. 2. The theoretical plot of dimensionless quantities for the sliding blocks. The solid line represents the behavior of the blocks, with an acceleration ratio of 1 below the critical point, and an increasing acceleration ratio [according to Eq. (4)] above the critical point. The transition from static to kinetic friction with greater force ratio is clearly not continuous. The gray region with an acceleration ratio less than 1 is not physically realizable, which contains the exam parameter set under the assumption of kinetic friction (circular marker).

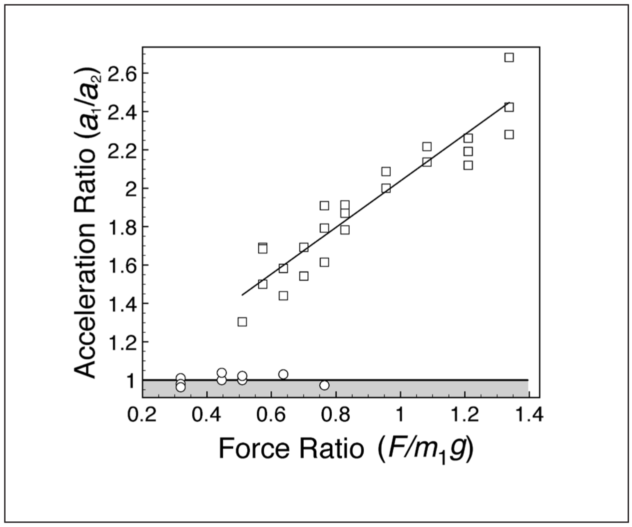

Fig. 3. Experimental results from video analysis of 36 trials. The mass ratio was constant at 0.40, while the hanging weight was varied. Two domains of data are distinguished: the blocks accelerated together in the static friction case (marked by circles), and the blocks separated with kinetic friction (squares). The line fit to the kinetic friction data is based on linear regression; the line at an acceleration ratio of one and the gray area are added for reference, as in Fig. 2.

and kinetic cases.

In designing the experiment, and especially while we thought about how to analyze and interpret the data, questions arose: How much force do we need? How do the blocks' accelerations compare? It became clear that the answers could be framed in terms of ratios: the *force ratio* compares the applied force to the weight of the lower block, F/m_1g, and the *acceleration ratio*, a_1/a_2. The acceleration ratio is particularly helpful for analysis and interpretation. At unity, the blocks stay together through static friction, while a ratio greater than 1 indicates the lower block sliding out from under the upper. Counterintuitive values for acceleration ratios indicate the accelerations would be in different directions (for a negative ratio), or the top block accelerates faster than the bottom (if the ratio were positive but less than one).

The choice of the type of friction gives different expectations based on theory as well as intuition. If static friction is the appropriate choice, the blocks will move together with the same acceleration. A quick derivation from Newton's second law yields Eq. (1),

$$a_1 = a_2 = \frac{F}{m_1 + m_2}. \tag{1}$$

In the case of kinetic friction, the interaction that accelerates the upper block will depend on the coefficient of kinetic friction. The accelerations are given in Eqs. (2) and (3):

$$a_1 = \frac{F - \mu_k m_2 g}{m_1} \tag{2}$$

$$a_2 = \mu_k g. \tag{3}$$

Dividing Eq. (2) by Eq. (3) leads to a single relationship in terms of the dimensionless ratios inspired by the experiment and analysis:

$$\frac{a_1}{a_2} = \frac{1}{\mu_k}\left(\frac{F}{m_1 g}\right) - \frac{m_2}{m_1}. \tag{4}$$

Treating the force ratio as the independent variable and the resulting ratio of accelerations as a dependent variable, Eq. (4) describes a line. The slope is the reciprocal of the friction coefficient, and the intercept is the negative mass ratio.

Figure 2 contains a plot of Eq. (4). Using this dimensionless plot, one can tell the story of the demonstration experiment: starting with an applied force that is small compared to the weight of the lower block, static friction controls the behavior of the blocks. They accelerate together, so even though the blocks' acceleration increases with increased force, the relative acceleration remains at unity. At some point, the static friction force $f_s \le \mu_s N$ reaches its limit. (N is the magnitude of the normal contact force.) Newton's second law for m_1, combined with the acceleration from Eq. (1), provides the force ratio at this critical point:

$$\frac{F}{m_1 g} = \mu_s \left(\frac{m_2}{m_1} + 1 \right). \qquad (5)$$

Greater applied force brings the system into the region of kinetic friction in which increasing force makes the lower block accelerate increasingly more than the upper block. Ultimately, the behavior of the blocks recalls the magician's trick of pulling the tablecloth from under the dishes; the lower block slides quickly from beneath the upper one.

The Results

The measurements collected by the students are presented in Fig. 3 using the same dimensionless format as Fig. 2. As the force is increased relative to the weight of the lower block, the static friction (data points marked with circles) and kinetic friction (squares) domains are evident, with a discontinuity near a force ratio of 0.5. Using this rough estimate of the critical point in Eq. (5), the coefficient of static friction between these blocks would be 0.36. Alternatively, we may use the maximum force ratio that demonstrated static friction, 0.76, which gives a coefficient of static friction of 0.54.

The slope of the kinetic friction data was found by linear regression to be 1.21 ± 0.09. From its inverse, the coefficient of kinetic friction is 0.83—unexpectedly greater than the coefficient of static friction! With the shallow slope of the regression line fitted to the kinetic friction data, its y-intercept is high: 0.83 ± 0.08 rather than the -0.40 expected with this mass ratio. Finally, several data distinguished as static in Fig. 3 have acceleration ratios slightly less than 1. This is attributed to experimental error involved in numerical differentiation of position data; it is *not* evidence that the top block can ever accelerate faster than the lower block!

The most intriguing aspect of the experimental results was the transition to sliding friction; it was not as sharply defined as the theory might suggest. For three different hanging masses, trials showed split results—some runs demonstrated static friction and others kinetic. A quick review of the literature[2-4] reveals that friction is not as straightforward as some students might initially infer from textbooks, and determining the coefficients of this friction model can require some care and diligence. The Coulomb model itself may need to be refined or abandoned. As with most topics in an introductory course, much deeper investigation could be pursued.

Conclusions

This investigation explored how nature "chooses" between static and kinetic friction in a situation based on a classic textbook problem of sliding blocks. The dimensionless plot is a powerful visual device to capture the dynamics of the sliding blocks, as it provides a very general way to tell the story of the phenomena without recourse to specific parameter values. The plot incorporates both the static case, with blocks moving together, and the kinetic case, in which the lower block slides from under the upper block. Additionally, the plot facilitates the analysis of experimental data to determine both coefficients of static and kinetic friction. The coefficient of static friction is found from the critical point, and that of kinetic friction is determined from the slope of the line of best fit to the kinetic friction data. Results from our experiment support the visual presentation of the simple Coulomb model of friction while suggesting a more careful approach to both experiment and modeling at the critical point between static and kinetic friction.

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References

1. P.A. Tipler, *Physics for Scientists and Engineers,* 3rd ed.(Worth Publishers, 1990).
2. J. Ringlein and M.O. Robbins, "Understanding and illustrating the atomic origins of friction," *Am. J. Phys.* **72,** 884–891 (July 2004).
3. P.S. Carvalho and A.S. e Sousa, "An inexpensive technique to measure coefficients of friction with rolling solids," *Phys. Teach.* **43**, 548–550 (Nov. 2005).
4. M. Kinsler and E. Kinzel, "A simple lab exercise to determine the coefficient of static friction," *Phys. Teach.* **44**, 77–79 (Feb. 2006).

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