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## A Simple Electric Field Probe in a Gauss's Law Laboratory

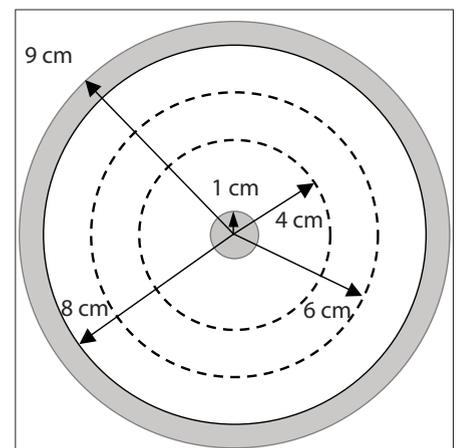
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Early in our calculus-based introductory course, students are introduced to electric fields and sometimes struggle with the abstraction of a vector field. They have less familiarity with the phenomena associated with electric fields, and the connection between phenomena and mathematical formalism is weaker.<sup>1</sup> Our very next topic is Gauss's law. While this provides an elegant approach to finding electric field in special cases, the expression involves additional mathematical baggage that may further obscure the electric field. These students aren't yet comfortable with dot products and surface integrals. The usual symmetries are invoked in example problems but require students to visualize flux in three dimensions. It is possible to reach this point in the introductory sequence without venturing outside of two-dimensional problems.

To address these weaknesses with Gauss's law and visualizing flux, we have adapted familiar laboratory exercises using conducting paper.<sup>2</sup> In these exercises (several examples are available online)<sup>3–6</sup> students measure potential at various points in a two-dimensional space containing charged conductors. Then they connect these points with the same

potential to create equipotential “surfaces” (lines in the two-dimensional space on the paper). Perpendicular to the equipotential lines, they can draw qualitative electric field lines. To reinforce the quantitative relationship between electric field and electric potential, students can also calculate the average electric field between two points: the ratio of potential difference to the distance between the points. Taking a pair of points in the radial direction, in an example with concentric conductors, one can find the component of electric field in the radial direction and compare it to theoretical prediction.<sup>7</sup> Computers can assist in this effort, especially in processing the data to create contour plots or three-dimensional surfaces in perspective.<sup>8</sup>

For a first experience with electric field in the laboratory, however, electric potential has not yet been introduced. Students are comfortable with the digital voltmeter, so we use electrodes with a fixed distance of one centimeter to measure the vector electric field on conducting paper. We do not mention the calculation with difference in potential, but only state that the voltmeter reads electric field in units of volts per centimeter. Then a simple conversion to the



**Fig. 1. A diagram of the concentric circles applied to the conducting paper. Gray shaded areas show electrodes of silver paint, and the concentric circles for flux calculations and Gauss's law are shown with dashed lines.**

more familiar N/C is provided:  
1 V/cm  $\rightarrow$  100 V/m  $\rightarrow$  100 N/C.  
The exercise is written to engage students in direct measurement of the vector electric field and to give them a way to explore Gauss's law before learning about electric potential. A follow-up laboratory exercise also uses the conducting paper to relate electric potential and field, and at that point students develop a more complete understanding of how the electric field is measured on conducting paper.<sup>9</sup>



**Fig. 2.** The experimental apparatus features the electric field probe: a small block of wood with two electrodes made from upholstery pins. For scale, the grid spacing on the conducting paper is 1 cm. The power supply (Heath 2732) and multimeter (Metorman 38XR) are standard lab equipment.

## Laboratory Equipment

The equipment involved in this exercise is similar to the PASCO Field Mapper Kit. A standard lab power supply provides charge separation between conductors of silver paint on conducting paper. Our conducting paper is prepared with concentric circles of conducting paint. A solid circle in the center has a radius of roughly 3 cm; the outer circle is located at about  $r = 10$  cm, as in Fig. 1. For our laboratory, these are meant to represent a cross-sectional slice of a coaxial cable.

The simple electric field probe was designed to provide a known distance between two electrodes, carefully set to 1.0 cm. These electrodes must make good contact with the paper. The probe we use is assembled from a small block of wood and two upholstery pins with smooth round ends to preserve the integrity of the paper. The probe is pictured in Fig. 2.

Students determine both magnitude and direction of the electric field using the probe. They are instructed to fix the location of the pin

connected to the voltmeter ground, then “walk” the other pin (connected to the +V port) in a circle around this point until the voltmeter reading is most *negative*. We have found that students adapt readily to this procedure, which is comparable in difficulty to orienting a Hall probe (the Hall element is perpendicular to the magnetic flux when the reading is greatest). The direction from the fixed, grounded pin to the other pin is the direction of the electric field. As we like to tell the students, the electric field points “downhill”—toward lower potential, so we look for the most negative voltmeter reading. The absolute value of that reading is taken to be the magnitude of the average field between the pins.

## Using the Probe to Discover Gauss’s Law

We first ask students to investigate the electric field between the concentric conductors. They are readily able to determine that the electric field is radial, directed away from the center electrode connected to the positive terminal of the power supply. When asked to measure the strength of the field at several locations equidistant from the center, they find that the magnitude does not vary significantly. This is in agreement with the cylindrical symmetry of the system.

We then take the opportunity to use this system to lead the students to “discover” Gauss’s law. Students measure the electric field at several equally spaced points around a circle, which lies between the electrodes. Since the definition of flux involves a surface in three-dimensional space, we ask the students to imagine extending the circle perpendicular to the plane of the paper a distance

$L$ . (Our engineers see that this “extruded” circle is a cylinder.) They can then derive the flux  $\Phi$  of the field through this circular cylinder of area  $2\pi rL$ :

$$\Phi = 2\pi rL \langle E \rangle, \quad (1)$$

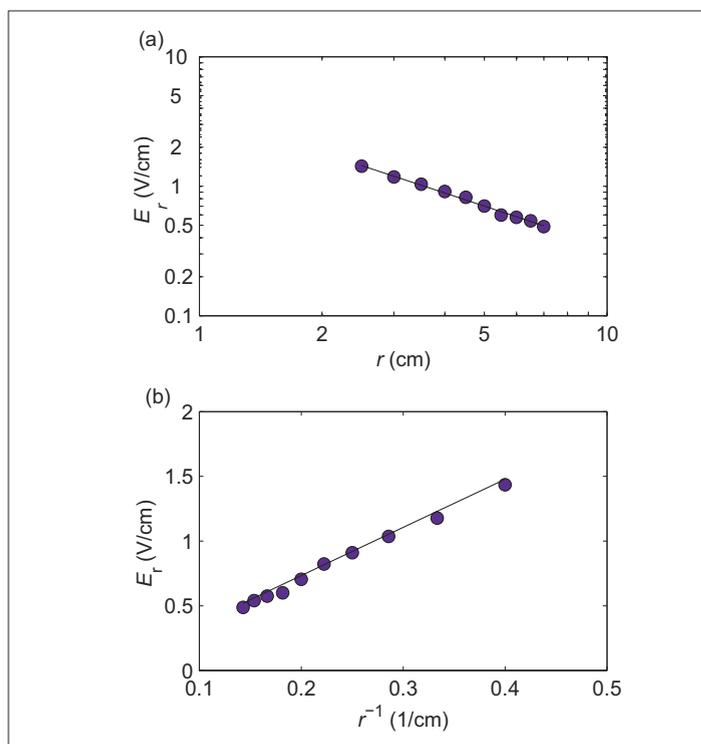
where  $\langle E \rangle$  is the average field for this circle of radius  $r$  and  $L = 1$  m for easy calculation. The field measurements are repeated for a circle of a different radius, and the calculation of the flux is repeated. The two values of flux are compared to see if they agree within experimental uncertainty (propagated from the uncertainty in the measurement of  $r$  and the uncertainty in  $\langle E \rangle$ , taken as the standard deviation of the mean). The heart of Gauss’s law relates flux to enclosed charge; both circles enclose the same charged electrode, so we expect the flux through both cylinders is equal.

## Using Gauss’s Law to Find Electric Field

Finally, the coaxial cable is a standard example in lecture, as cylindrical symmetry can be used with Gauss’s law to find the electric field between the conductors. The result,

$$E_r = \frac{\lambda}{2\pi\epsilon} \frac{1}{r}, \quad (2)$$

predicts the radial electric field component  $E_r$  should depend on the inverse of the radial distance. The effective linear charge density  $\lambda$  is related to the charge on the center electrode. In the last part of the lab, students derive this expression, make several measurements of electric field along a radial line, and use logarithmic plots of  $E_r$  vs  $r$  to verify the inverse-



**Fig. 3. Two presentations of typical data. (a)  $E_r$  vs  $r$  plotted on logarithmic axes to verify the dependence on  $r^{-1}$ . The slope of the fit is  $-1.04 \pm 0.02$ . (b)  $E_r$  vs  $1/r$  with a best-fit line to derive the effective linear charge density. The slope of the line is  $3.72$  V, and the intercept is  $-0.01$  V/cm.**

distance relationship. A plot of  $E_r$  vs  $1/r$  provides a value for the effective linear charge density  $\lambda$  on the central electrode. Examples of student data are shown in Fig. 3.

## Conclusion

The probe with electrodes at a fixed separation allows students to measure electric field directly on two-dimensional conducting paper. An accessible extension of this into three dimensions leads students to calculate electric flux from these measured electric fields. They develop a conceptual understanding of Gauss's law as they relate electric flux to enclosed source charge. Finally, students apply Gauss's law to predict electric field within a coaxial cable. Throughout, the vector nature of the electric field

is emphasized: orientation and magnitude are both important. While we avoid the traditional calculation using electric potential, this laboratory serves as a natural introduction to the potential mapping experiment later in the course.

## References

1. D.P. Maloney, T.L. O'Kuma, C.J. Hieggelke, and A. Van Heuvelen, "Surveying students' conceptual knowledge of electricity and magnetism," *Am. J. Phys.* **69**, S12-23 (July 2001).
2. Field Mapper Kit (PK-9023) from PASCO scientific, 10101 Foothills Blvd., Roseville, CA 95678-9011 or <http://www.pasco.com>.
3. "Lab 3: Electric Fields II" combines a fixed-separation probe and potential mapping with software simulation

to explore electric field and flux at [http://www.umd.umich.edu/cas/natsci/faculty/matzke/Lab\\_M126-3.PDF](http://www.umd.umich.edu/cas/natsci/faculty/matzke/Lab_M126-3.PDF).

4. "The Field of an Electric Dipole" also uses a fixed-separation probe for the electric field at [http://it.stlawu.edu/~physics/labs/152\\_lab/efield.pdf](http://it.stlawu.edu/~physics/labs/152_lab/efield.pdf).
5. "Lab 2 – Potential Plotting" uses two probes with handles held together to find electric field at <http://www.dartmouth.edu/~physics/labs/p16/lab2.pdf>.
6. "Electric Fields and Potentials" employs a spacer that accepts the two probes at a fixed distance to find electric field at [http://www.phy.olemiss.edu/~thomas/weblab/215\\_lab\\_items/20\\_215\\_Exp8\\_Electric\\_Poten.pdf](http://www.phy.olemiss.edu/~thomas/weblab/215_lab_items/20_215_Exp8_Electric_Poten.pdf).
7. Martha Lietz, "A Potential Gauss's law lab," *Phys. Teach.* **38**, 220–221 (April 2000).
8. R.A. Young, "Quantitative experiments in electric and fluid flow field mapping," *Am. J. Phys.* **69**, 1223–1230 (Dec. 2001).
9. The conducting paper, of course, is not truly electrostatic. A small trickle of charge flows between electrodes through the distributed resistance provided by the paper. As long as the paper is reasonably homogeneous, this subtlety is easy to neglect. However, the analogy is occasionally brought to light when reused paper yields unexpected results at well-used locations; students are then encouraged to think about how the properties of the paper medium affect the electric field within it. [The authors thank their reviewer, who mentioned that the apparatus is merely an analogy of the electrostatic field.]

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